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Equilibrium States for a Plane Incompressible Perfect Fluid

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Abstract. We associate to the plane incompressible Euler equation with periodic conditions the corresponding Hopf equation, as an equation for measures on the space of solenoidal distributions. We define equilibrium states as the solutions of the stationary Hopf equation. We find a class of equilibrium states which corresponds to a class of infinitely divisible distributions, and investigate the properties of gaussian and poissonian states. Equilibrium dynamics for a class of poissonian states is constructed by means of the Onsager vortex equations.

1. Introduction

The purpose of this paper is to exhibit a class of equilibrium states for a plane fluid which moves according to the incompressible Euler equation, and to study their main properties. Our treatment will be limited to the case of periodic boundary conditions, which allows an explicit use of Fourier methods.

Equilibrium states for the plane incompressible Euler fluid have been studied by physicists for a long time. Among the most significant contributions we may mention a paper by Lee [1] in which a class of gaussian states were introduced as macrocanonical equilibrium states corresponding to energy and enstrophy conservation, and a paper by Novikov [2] in which equilibrium states corresponding to poissonian distribution of vortices are studied.

The mathematical definition of equilibrium state which we give is based on the Hopf equation associated to the plane incompressible Euler equation, namely we define equilibrium states as solutions of the stationary Hopf equation. We recall that the Hopf equation describes the evolution of measures on phase space associated to the point evolution, and is written in terms of the characteristic functionals.

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