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# Continuation of Partial-Wave Two-Cluster Atomic Scattering Amplitudes*

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**Abstract.** It is shown, with some restrictions, that on-shell two-cluster partial-wave scattering amplitudes for atomic systems whose particles interact via two-body Coulomb potentials have analytic continuations in the complex energy plane below the physical part of the real axis. The result is proved only for energies lower than any three (or more) cluster threshold. Poles of the amplitudes can occur only at discrete eigenvalues of the rotated Hamiltonian which may be reached by continuation along the same path. The method of proof uses analyticity related to a generalized scaling transformation and the boost transformation.

## 1. Introduction

In a previous paper [1], referred to as I, we studied an $N$-particle atomic system whose Hamiltonian with center of mass part removed is $H$. We concentrated on the range of energy below $E_{\min}$ the lowest threshold of all those corresponding to three or more bound clusters, so that only channels consisting of two bound clusters are open, and, to simplify the argument, also assumed that at least one of each pair of clusters was neutral. We proved I Theorem 1 which showed that the limit

$$T_{fi}(E) = \lim_{\varepsilon \downarrow 0} (\psi^f, V^f G(E + i\varepsilon) V^i \psi^i) \tag{1}$$

exists for $E$ in the above range on the complement of a closed set of measure zero. In (1) $\psi^i, \psi^f$ describe the two-cluster states in channels $i, f$ respectively and each correspond to a given partial wave, and $V^i$, $V^f$ are respectively the sums of the two-body potentials between all pairs of particles in different clusters of $i, f$. We also use $G(z) = (Z - H)^{-1}$.

The effect of I was thus to show that, under the above conditions, the conventional time-independent formula for the scattering amplitude, $T_{fi}(E)$ + Born term, makes sense. In this paper we show that $T_{fi}(E)$ (and thus the full amplitude) may be

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