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Singular Points and Projective Limits in Relativity

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Abstract. We describe a modification of Schmidt's b-boundary for a space-time, using a projective limit construction. The resulting boundary provides endpoints for all incomplete inextensible curves that are not totally or partially trapped, and every boundary point is an endpoint of such a curve. Boundary points are always Hausdorff separated from interior points, and the construction gives separate past and future singularities in the $k=+1$, $\Lambda=0$, Friedmann cosmology.

1. Introduction

The singularity theorems of Hawking and Penrose [1] have made it clear that general relativistic models of physically reasonable situations (e.g. of collapsing stars or the matter filled universe) are likely to contain timelike or null curves that cannot be continued to arbitrarily large or negative values of proper time (or affine parameter). The word "singularity" was used to denote this state of affairs, which corresponds physically to the strange idea of particles or photons suddenly ceasing to exist (or coming into being). One feels that there should be some "reason" for such a situation, and so, in view of the properties of known exact solutions, one is led to conjecture that in some sense the curvature of space-time becomes unbounded along such a curve, the unboundedness leading to a breakdown of general relativistic physics; or more generally it could be that the curvature becomes unbounded arbitrarily near the curve in some sense.

A way of mathematically formulating this idea is to define singular points, attached to the space-time of general relativity as a topological boundary, so that one can talk about the behaviour of physical quantities, such as curvature, in the limit as one approaches a singular point. Various definitions of such boundaries have been proposed, the most natural mathematically being the bundle boundary (b-boundary) of Schmidt [2]. The precise definition is given below, but roughly speaking, it attaches to a space-time M a (topological) boundary $\partial_b M$ which provides an endpoint for each curve in M that is incomplete in a generalised sense;

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