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BPHZL Renormalization of $1/N$ Expansion and Critical Behaviour of the Three-Dimensional Chiral Field

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Abstract. A modified soft mass renormalization scheme for $1/N$ expansion of the three dimensional $O(N)$-invariant chiral field in the high- and low-temperature phases, as well as at the critical point, is constructed, free of infrared divergencies in each separate diagram. Generalized quantum chirality identities for composite operators are derived, from which the renormalizability of the model follows. The approach formulated here is applied to a rigorous analysis of the universal critical behaviour of the N-component chiral field in three dimensions.

1. Introduction

In this paper we consider Euclidean chiral field $n(x)=(n_1(x), \ldots, n_N(x))$, $x \in \mathbb{R}^3$ taking values on the sphere with the simple Lagrangian:

$$\mathscr{L} = -\tfrac{1}{2}(\partial_\mu n)^2 , \tag{1a}$$

$$n^2(x) = \frac{N\mu}{T}, \tag{1b}$$

where $n^2(x) \equiv (n(x), n(x)) = \sum_{\alpha=1}^{N} n_\alpha^2(x)$, [$\mu$: mass parameter, T: coupling constant (temperature)].

The chiral field appears in various problems in different (Euclidean) spacetime dimensions D. In statistical physics it describes the critical behaviour of classical lattice Heisenberg model of N-component spins with $O(N)$ invariant intersection [1]. In field theory it is used in a series of models in the realistic case $D=4$, $x \in \mathbb{M}^4$. The chiral field was first introduced [2] to describe the low-energy π-meson scattering (non-linear σ model). It was applied to construct an unfield model of weak and electromagnetic interactions [3], to introduce mass of the Yang-Mills field in a gauge invariant manner [4], and in models of extended particles [5–7]. Recently there has been an increased interest in the two-dimensional chiral field model ($x \in \mathbb{M}^2$ or $\mathbb{R}^2$) due to the existing similarities between it and non-Abelian

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