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# Construction of Quantised Gauge Fields

## II. Convergence of the Lattice Approximation

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## I. Introduction

We continue in this paper a program initiated in [1], henceforth referred to as Paper I. One of the objectives set forth in that paper was a mathematically complete construction of a super-renormalisable continuum gauge theory. This paper contains results in this line of work.

The study of gauge theories on a lattice was originally suggested [2] as a suitable starting point for learning more about gauge theories generally, because lattice gauge theories provide a setting in which one can utilise methods of statistical mechanics: – low and high temperature expansions and correlation inequalities, etc. In addition these theories possess the two important properties of Osterwalder-Schrader positivity and gauge invariance. No other method, yet proposed, of regularizing continuum gauge theories so that they become mathematically well-defined objects possesses all these attractive features. It is therefore an important problem to verify that these theories converge in a suitable sense to continuum theories when the lattice spacing is taken to zero. The limit would then share these properties and in addition one would hope to verify that it is Euclidean invariant (unlike the lattice theories). Various consequences of the correlation inequalities which will be of interest to physicists as well as mathematicians have been outlined in [3].

Unfortunately, it is unlikely that our method of proving convergence is optimal. We have adopted a method of embedding lattice gauge theories in continuum theories which is not natural in the context of geometry. It might be rewarding to search for methods that treat the geometrical side with less than the insensitivity that we have been able to muster. In the meantime we have in this paper a number of functional analytic techniques that will extend to more singular theories, abelian and non abelian and some of them will very likely be useful in future improvements.

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