Commun. Math. Phys. 71, 95—130 (1980)

Communications in
Mathematical
Physics

Ultraviolet Stability in Euclidean Scalar Field Theories

G. Benfatto, M. Cassandro, G. Gallavotti*, F. Nicoló, E. Olivieri, E. Presutti, and E. Scacciatelli

Istituti di Matematica e Fisica dell'Università, Rome, Italy

Abstract. We develop a technique for reducing the problem of the ultraviolet divergences and their removal to a free field problem. This work is an example of a problem to which a rather general method can be applied. It can be thought as an attempt towards a rigorous version (in 2 or 3 space-time dimensions) of the analysis of the structure of the functional integrals developed in [9], the underlying mechanism being essentially the same as in [11, 3].

1. Introduction

The free euclidean field in $\mathbb{R}^d$ is the gaussian field with covariance operator:

$$C=(1-D)^{-1}, \tag{1.1}$$

where D is the Laplace operator in $\mathbb{R}^d$.

We call $(\varphi_\xi)_{\xi\in\mathbb{R}^d}$ the random field with the above covariance and we shall represent φ as a sum of independent, identically distributed up to scale factors, random fields $(\varphi_\xi^{(N)})_{\xi\in\mathbb{R}^d}$, $N=0,1,\ldots$. The field $\varphi^{(N)}$ has, by definition, the following covariance operator:

$$C^{(N)}=(\gamma^{2N}-D)^{-1}-(\gamma^{2N+2}-D)^{-1} \tag{1.2}$$

and γ will be appropriately chosen close to 1. From (1.2) it follows that if $d\leqq 3$ the kernel $C_{\xi\eta}^{(N)}$ of $C^{(N)}$ in $\mathbb{R}^d$ is finite when $\xi=\eta$:

$$C_{\xi\xi}^{(N)}\equiv C_{00}^{(N)}=\gamma^{(d-2)N}\frac{\gamma^2-1}{(2\pi)^d}\int\frac{d^dk}{(1+k^2)(\gamma^2+k^2)}\equiv c_\gamma\gamma^{(d-2)N}. \tag{1.3}$$

Hence it will be convenient to introduce the normalized field:

$$z_\xi^{(N)}=\frac{\varphi_\xi^{(N)}}{\sqrt{2C_{\xi\xi}^{(N)}}}=\frac{\varphi_\xi^{(N)}}{\sqrt{2c_\gamma\gamma^{(d-2)N}}} \tag{1.4}$$

---

* Supported by IHES, through financial support from the "Stiftung Volkswagenwerk"

0010-3616/80/0071/0095/$07.20