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On the Relation Between Classical and Quantum Observables

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Abstract. For systems with a finite number of degrees of freedom, the relation between classical and quantum observables is analysed. In particular, a precise statement of the correspondence limit is obtained.

1. Introduction

Consider a physical system with n degrees of freedom. Let us suppose that in the (non-relativistic) classical description, one can introduce a smooth, n-dimensional, orientable manifold **C** (without boundary) to represent the configuration space of the system. Then, the cotangent bundle Γ over **C** represents the phase space. This Γ serves as the arena for classical mechanics: points of Γ describe the permissible classical states while suitably regular functions on Γ describe the classical observables. Dynamics manifests itself via a Hamiltonian vector field. The space of classical observables is endowed with two interesting mathematical structures. The first of these is the structure of an associative Abelian algebra, available on the space of functions on any manifold, while the second is the structure of a Lie algebra induced by the Poisson bracket which itself arises due to the presence of a natural symplectic structure on Γ.

In order to "quantize" this system, there is available the Dirac prescription [1]: Replace the Poisson brackets between classical observables by ($\hbar/i$ times) the commutators of the corresponding quantum observables. Although the intuitive idea underlying this prescription is transparent, the precise "method of replacement" is not. What does "corresponding quantum observables" mean? Indeed, the well known factor ordering problems arise precisely because classical observables do not, in general, have unambiguous quantum analogues. For the class of systems described above, is there perhaps a universally available set of classical observables which does have its quantum analogue? More generally, what is the precise relation between classical and quantum observables? In the classical limit, does the commutator between two quantum operators vanish or is it related to the Poisson bracket between the corresponding classical observables? Is this cor-

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