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Integral Decomposition of Unbounded Operator Families

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Abstract. We give a meaning to the direct integral decomposition of unbounded operators and Op*-algebras on a metrizable dense domain of a Hilbert space, by considering them as bounded operators between several other Hilbert spaces.

Introduction

The decomposition of representations and states of $*$-algebras into irreducible representations and extremal states has been considered by Borchers and Yngvason [1] and Hegerfeldt [2] in the important case of nuclear $*$-algebras. The method of [2] consists in restricting the state to a dense subalgebra which is the finite linear span of countably many elements. Since the algebraic dual of such a subalgebra has a proper, metrizable and weakly complete positive cone, Choquet decomposition theory [3] can be applied to it. The restriction of the state is thus decomposed into extremal states of the subalgebra and these are continuous states (because the initial algebra is nuclear) and so can be extended to the whole algebra.

In this paper we want to consider non nuclear $*$-algebras and if we try the method of [2] we are not able to extend the extremal states of the subalgebra to the whole algebra (unless our algebra is such that every state on it is continuous; sufficient conditions for that are given in [4] p. 228). For that reason, we prefer to adopt the point of view of Borchers and Yngvason and to decompose first certain families of unbounded operators on a Hilbert space, the so-called Op*-algebras [5].

As usual for unbounded operators we have to distinguish between two different notions of commutant, the strong and the weak. A state is extremal if and only if the corresponding GNS-representation has a trivial weak commutant [6]. In the first part of [1] Borchers and Yngvason developed an extension theory for $*$-invariant families of unbounded operators. They showed that any such family $\mathscr{A}$ always has an extension $\hat{\mathscr{A}}$ such that its strong commutant $\hat{\mathscr{A}}'_s$ contains an Abelian von Neumann algebra $\mathscr{M}$ which is at the same time maximal Abelian in the weak commutant $\hat{\mathscr{A}}'_w$ (in order to get the irreducibility of the decomposition which will be performed with respect to $\mathscr{M}$). This extension theory is valid for any Op*-algebra (the nuclearity assumption comes only in the second part of [1]) so we are going to use it as well in our framework.

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