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Linear Spin-Zero Quantum Fields in External Gravitational and Scalar Fields

II. Generally Covariant Perturbation Theory*

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Abstract. The quantum theory of both linear, and interacting fields on curved space-times is discussed. It is argued that generic curved space-time situations force the adoption of the algebraic approach to quantum field theory: and a suitable formalism is presented for handling an arbitrary quasi-free state in an arbitrary globally hyperbolic space-time.

For the interacting case, these quasi-free states are taken as suitable starting points, in terms of which expectation values of field operator products may be calculated to arbitrary order in perturbation theory. The formal treatment of interacting fields in perturbation theory is reduced to a treatment of "free" quantum fields interacting with external sources.

Central to the approach is the so-called two-current operator, which characterises the effect of external sources in terms of purely algebraic (i.e. representation free) properties of the source-free theory.

The paper ends with a set of "Feynman rules" which seems particularly appropriate to curved space-times in that it takes care of those aspects of the problem which are specific to curved space-times (and independent of interaction). Heuristically, the scheme calculates "in-in" rather than "in-out" matrix elements. Renormalization problems are discussed but not treated.

Introduction

0.1. Motivation

There has recently been some interest in the problem of self, or mutually *interacting* quantum fields in curved space-times (see [1] and references therein). The value of this work is two-fold. Firstly, it is important to know just how the many recent results on *linear* quantum fields (see Sect. 0.2.) in curved space-times get modified in the more realistic case of interaction. Secondly, Einstein's (and other) theories

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