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Equilibrium States of Gravitational Systems

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Abstract. We formulate the equilibrium correlation functions for local observables of an assembly of non-relativistic, neutral gravitating fermions in the limit where the number of particles becomes infinite, and in a scaling where the region Ω, to which they are confined, remains fixed. We show that these correlation functions correspond, in the limit concerned, to states on the discrete tensor product $\bigotimes_{x\in\Omega}\mathscr{A}_x$, where the $\mathscr{A}_x$'s are copies of the gauge invariant C^*-algebra $\mathscr{A}$ of the CAR over $L^2(\mathbf{R}^3)$. The equilibrium states themselves are then given by $\bigotimes_{x\in\Omega}\bar{\omega}_{\varrho_0(x)}$, where $\bar{\omega}_\varrho$ is the Gibbs state on $\mathscr{A}$ for an infinitely extended ideal Fermi gas at density ϱ, and where ϱ_0 is the normalised density function that minimises the Thomas-Fermi functional, obtained in [2], governing the equilibrium thermodynamics of the system.

1. Introduction

The thermodynamical limiting behaviour of a non-relativistic assembly of N neutral, gravitating fermions of one species, confined to a suitably regular bounded three-dimensional domain Ω, is not of the usual type, since the internal energy, temperature and volume of the system scale like $N^{7/3}$, $N^{4/3}$ and N^{-1}, respectively, as $N\to\infty$ [1–4]. The system also possesses simple properties of scale invariance. In the particular scaling where the domain Ω and the temperature are fixed, while the particle mass and gravitational constant become proportional to $N^{2/3}$ and N^{-1}, respectively, the specific free energy tends, as $N\to\infty$, to the minimum value of the Thomas-Fermi functional Φ_0 on the bounded probability densities on Ω, given by the formula

$$\Phi_0(\varrho)=\int_\Omega d^3x\varphi_0(\varrho(x))-\tfrac{1}{2}\int_{\Omega^2} d^3xd^3y\frac{\varrho(x)\varrho(y)}{|x-y|}, \tag{1.1}$$

where $\varphi_0(\varrho)$ is the equilibrium free energy density of an ideal Fermi gas at density ϱ and at the given temperature, T. According to a numerical solution of the resultant

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