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Some Results for the Exponential Interaction in Two or More Dimensions

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Abstract. We show that for the regularized exponential interaction $\lambda :e^{\alpha\varphi}:$ in d space-time dimensions the Schwinger functions converge to the Schwinger functions for the free field if $d>2$ for all α or if $d=2$ for all α such that $|\alpha|>\alpha_0$.

1. Notations and Results

In this paper we study the space-time cut-off exponential interaction in d space-time dimensions $V_\Lambda=\lambda\int\limits_\Lambda :\mathrm{e}^{\alpha\xi(x)}:dx$, where Λ is a bounded subset of $\mathbb{R}^d$, $\lambda>0$ and the corresponding Euclidean measure $d\mu_\Lambda(\xi)=Z_\Lambda^{-1}e^{-V_\Lambda(\xi)}d\mu_0(\xi)$, μ_0 being the free Euclidean field of mass 1 on $\mathbb{R}^d$ [1], $\alpha\in\mathbb{R}$ and $: :$ being the Wick ordering (see below for details on notation). Such models of quantum fields were introduced in [2], and in [2, 3] it was shown that if $d=2, |\alpha|<\sqrt{4\pi}$ then $V_\Lambda\in L_2(d\mu_0)$ and μ_Λ is a (non Gaussian) probability measure. The existence of a measure μ_Λ of the above form was shown for all $d\geqq 2$ and arbitrary α in [4], see also [5] and, for a different proof, [6][1]. In [5] it was shown that in the case $d\geqq 4$ the regularized (ultraviolet cut off) version of the measure μ_Λ converges as the regularization is removed to μ_0. In the present paper we tackle, using a modification of the basic idea of [5] together with methods of [7], the case $d\geqq 3$ and also the case $d=2$ for $|\alpha|$ large. The results of the present paper were announced in [10]. Let us now give the notations and state the results. We define the free field on $\mathbb{R}^d$ with ultraviolet cut off at distance γ^{-N}, $\gamma>1$, N a positive number, as the Gaussian field ξ_N

$$\xi_N(x)=\int A_N(x-y)\xi(y)dy\,, \qquad x\in\mathbb{R}^d\,, \tag{1.1}$$

where A_N is the kernel of the operator

$$A_N=\left(\frac{\gamma^{2N}}{\gamma^{2N}-\Delta}\right)^{\frac{1+k_d}{2}} \tag{1.2}$$

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1 Other references for the exponential interaction are e.g. [8, 9]

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