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Additive Conservation Laws in Local Relativistic Quantum Field Theory

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Abstract. We consider generators Q of symmetry transformations acting additively on asymptotic particle states according to (1.1). [This equation can be derived for Q defined as integral over a conserved local current!]. For simplicity, we consider only the case that all asymptotic fields are scalar. Assuming that elastic scattering occurs at least in an open subset of the scattering manifold we show that Q is at most a *linear* combination of generators of the Poincaré group and internal symmetries.

1. Introduction

A couple of investigations led to the conclusion that in a quantum field theory with sufficient interaction, there are no other symmetries apart from internal ones and those of the Poincaré group. Important contributions were [1] and [2]. In [1] the conclusion was obtained by assuming in particular the S-matrix to be analytic in the whole physical region and the symmetry generators to be self-adjoint. In [2], the conclusion was proven for a subclass of translation invariant generators.

In the present paper we take up this problem within the Wightman framework augmented by rather modest additional assumptions. Clearly, some kind of interaction assumption is necessary: It is known from examples that in theories without interaction there are many more generators than those mentioned above, see e.g. [5]. Furthermore, if generators are defined as integrals over conserved local hermitian current densities, there is an example showing that such a generator need not have self-adjoint extensions [5]. Hence, a hypothesis of self-adjointness should be avoided.

Considering a generator Q as an integral over a conserved local not necessarily covariant current density, we proved in [3, 4] that

$$i[Q, \psi_\kappa^{\mathrm{ex}}(x)] = P_{\kappa\lambda}(x, \partial)\psi_\lambda^{\mathrm{ex}}(x) \tag{1.1}$$

(Summation convention!). Here, $\psi_\kappa^{\mathrm{ex}}(x)$ ("ex" stands for "in" or "out") are free asymptotic fields, $P_{\kappa\lambda}$ are polynomials in $x \in \mathbb{R}^4$ and derivatives $\partial = \left(\frac{\partial}{\partial x^\nu}\right)$,

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