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Selfduality and Topological-Like Properties of Lattice Gauge Field Theories. A Proposal

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Abstract. We introduce for lattice gauge theories an analogue of the Pontrjagin index and a notion of "selfduality" and "antiselfduality". Selfdual and antiselfdual configurations on the lattice have much of the same properties (with some remarkable differences) as the corresponding configurations on the continuum, to which they converge when the lattice spacing goes to zero.

1. Introduction

In the past few years much effort has been put on the study of the topological properties of gauge fields. Particular emphasis has been placed on the use of topological invariants and of relevant techniques from differential and algebraic geometry to classify the solutions of the Yang-Mills equations which correspond to absolute minima of the action functional [1–8].

The ultimate interest in gauge theories from the point of view of elementary particle physics is of course the solution of the quantum Yang-Mills equations. The hope to be able to use also for gauge theories an approach trough (an Euclidean version of) path integral techniques makes it generally felt that one could benefit from a thorough understanding of the set of classical solutions, especially the absolute minima of the action, from a topological point of view.

The fact that the (yet to be constructed) measure on the space of connections (Yang-Mills potentials) is expected to give measure zero to the set of differentiable potentials (while the classical solutions are twice differentiable almost everywhere) is probably not a serious objection as one can see from the analogous problem in Quantum Mechanics.

On the other hand a mathematical theory of integration on function spaces does not exist at present for gauge theories; in view of the experience accumulated

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