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On the Bose-Einstein Condensation of an Ideal Gas

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Abstract. A mathematically precise treatment is given of the well-known Bose-Einstein condensation of an ideal gas in the grand canonical ensemble at fixed density. The method works equally well for any of the standard boundary conditions and it is shown that the finite volume activity converges and that in three dimensions condensation occurs for Dirichlet, Neumann, periodic, and repulsive walls.

1. Introduction

The phenomenon of Bose-Einstein condensation of an ideal gas is an elementary example of a phase transition. A rigorous discussion of this phenomenon within the framework of algebraic statistical mechanics was given by Araki and Woods [1] and has subsequently been developed by many authors [4–8]. It is our purpose here to present an account of Bose-Einstein condensation with particular attention given to the infinite volume limit of the grand canonical ensemble at *fixed density* with a careful proof of the convergence of the finite volume activities. The method we present here estimates the contribution of a *collection* of low lying one-particle energy eigenstates in contrast to the customary separation of the single ground state (e.g. [7] for the case of periodic boundary conditions). As a consequence, our method allows a unified and elementary treatment of a variety of boundary conditions, although our analysis will not prove that the condensate is asymptotically in a single eigenstate. We consider the same boundary conditions as discussed by Robinson [5] and complete his analysis by allowing the finite volume activity to vary. Indeed the condensation phenomenon cannot be entirely understood in terms of grand canonical ensembles with fixed activity. (Robinson [5] shows that Dirichlet boundary conditions do not lead to condensation if the activity z is fixed equal to one, but we show that if z is varied with volume so as to keep the average density fixed then condensation can occur at the limit value $z=1$. The Dirichlet case with variable z is also discussed in [8].) In three dimensions, condensation occurs irrespective of whether Dirichlet, Neumann, periodic or

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