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Large Orders in the $1/N$ Perturbation Theory by Inverse Scattering in One Dimension

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Abstract. When one tries to compute large orders in the $1/N$ series "à la Lipatov" a complicated non-linear equation for the instanton is found in ϕ^4 or non-linear sigma models.

We solve here this equation in the one-dimensional case (quantum mechanics) by inverse scattering techniques. From the instanton solutions we obtain the K^{th} order of the $1/N$ perturbation theory up to $0(K^{-1})$ for the $0(N)$ symmetric anharmonic oscillator and up to a factor $0(K^0)$ for a non-symmetric model. In the symmetric case we agree with results recently obtained in quantum mechanics by Hikami and Brézin following a different procedure. For the non-symmetric anharmonic oscillator we believe our formulae are new.

1. Introduction

In the last few years a great attention is paid to perturbative expansions in $1/N$ in quantum field theory, statistical mechanics and particle physics, N being the size of an internal symmetry group. However little is known in general about the nature of this expansion.

Typically, the N^{-1} series follow by expanding the functional integral in an appropriate representation around some constant stationary point (here labelled "0"). For example in the N-component ϕ_ν^4 theory with lagrangian

$$L=\frac{1}{2}(\partial_\mu\vec{\phi})^2+\frac{\mu^2}{2}\vec{\phi}^2+\frac{g\mu^{4-\nu}}{N}(\vec{\phi}^2)^2$$

the generating functional can be written as (see ref [1] and section II)

$$Z(N)=\int\int D\alpha(.)\exp(-S[\alpha(\cdot),N] \tag{1.1}$$

$$S=\frac{N}{2}\operatorname{tr}\log\left[-\nabla^2+\mu^2+4i\mu^{2-\nu/2}\sqrt{\frac{g}{N}}\alpha(\cdot)\right]+\int d^\nu x\alpha(x)^2. \tag{1.2}$$

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