Commun. Math. Phys. 70, 1–27 (1979)

Communications in
Mathematical
Physics

Scattering Theory and Dispersion Relations for a Class of Long-Range Oscillating Potentials*

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Abstract. If a spherically symmetric potential is such that $\int_r^{\to\infty} V(r')dr' = O(\exp - \mu r)$, and if an additional regularity condition is imposed [a sufficient one being that $rV(r)$ is L^1], the partial wave amplitudes are meromorphic in a strip of width μ in the complex momentum plane, and the full scattering amplitude is analytic inside an ellipse at fixed energy and satisfies fixed momentum transfer $(\sqrt{-t})$ dispersion relations for $|t| < \mu^2$.

Such a class of potentials includes not only exponentially decreasing potentials but also long-range oscillating potentials such as $(1+r^2)^{-2}\sin(\exp \mu r)$. In fact the results can partly be extended to a still broader class of potentials with increasing amplitude at infinity. It is argued that these results might lead to a revision of conventional ideas on what is the potential between physical hadrons.

Appendices may be of interest to special functions addicts.

I. Introduction

In this paper we propose to study non-relativistic scattering theory and dispersion relations for a class of spherically symmetric, long-range potentials which are very rapidly oscillating at large distances. As an example, consider the potential

$$V_1(r) = (1+r)^{-3}\cos(\exp(\mu r)) \tag{I.1}$$

This potential satisfies the condition

$$rV(r) \in L^1(0,\infty) \tag{I.2}$$

and it is well-known that for such potentials all the machinery of usual scattering theory, including the use of the Jost functions to define the S matrix and the bound

* Dedicated to Nick Khuri
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