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Solutions to the Ginzburg-Landau Equations for Planar Textures in Superfluid ^{3}He

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Abstract. The Ginzburg-Landau equations for planar textures of superfluid ^{3}He are proved to be equivalent to a completely integrable Hamiltonian system. General solutions to these equations are obtained by means of hyperelliptic integrals.

1. Introduction

Superfluid ^{3}He in the state of the p-pairing can be described in terms of a complex 3×3 matrix field A_{pi} (the order parameter), which minimizes the Ginzburg-Landau free energy, [1, 3, 5],

$$\begin{aligned} \mathscr{F} &= \int d^3x [F_{\mathrm{grad}} + F_b + F_h + F_d]; \\ F_{\mathrm{grad}} &= \gamma_1 \partial_K A^*_{pi} \partial_K A_{pi} + \gamma_2 \partial_K A^*_{pi} \partial_i A_{pK} + \gamma_3 \partial_K A^*_{pK} \partial_i A_{pi}; \\ F_b &= \alpha Tr(A^+ A) + \beta_1 |Tr(AA^t)|^2 + \beta_2 [Tr(A^+ A)]^2 \\ &\quad + \beta_3 Tr[(A^* A^t)(AA^t)^*] + \beta_4 Tr[(A^+ A)^2] + \beta_5 Tr[(A^+ A)(A^+ A)^*]; \end{aligned} \tag{1}$$

F_d is the dipole energy density,

F_h is the magnetic energy density.

For a uniform spacial configuration of the order parameter the F_{grad} terms are absent. Then the minimization of F_b gives values of the order parameter A_{pi} for the familiar A and B phases, which constitute smooth manifolds M_A and M_B.

In these two cases the order parameter is of the form:

(I) for the A phase

$$A_{pi} = \Delta \cdot d_p(\Delta'_i + \sqrt{-1}\Delta''_i), \qquad d^2 = 1, \qquad (\Delta'_i)^2 = (\Delta''_i)^2 = 1$$

$$\Delta = \mathrm{const}, \qquad \Delta'_i \cdot \Delta''_i = 0$$