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Z_4-Symmetric Factorized S-Matrix in Two Space-Time Dimensions

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Abstract. The factorized S-matrix with internal symmetry Z_4 is constructed in two space-time dimensions. The two-particle amplitudes are obtained by means of solving the factorization, unitarity and analyticity equations. The solution of factorization equations can be expressed in terms of elliptic functions. The S-matrix contains the resonance poles naturally. The simple formal relation between the general factorized S-matrices and the Baxter-type lattice transfer matrices is found. In the sense of this relation the Z_4-symmetric S-matrix corresponds to the Baxter transfer matrix itself.

1. Introduction

During the last years a number of examples were found of nontrivial and exactly calculable relativistic scattering theories in two space-time dimensions (see for the review [1] and references therein). These examples, the so-called factorized S-matrices, correspond to simplified scattering kinematics restricted by special selection rules. These forbid a change in the number of particles and also preserve the set of individual momenta. Therefore the scattering process reduces to the redistribution of momenta between the different particles of the same mass. Selection rules of this type are characteristic of the quantum dynamics of a completely integrable field-theoretic system such as the sine-Gordon model.

The presence of these selection rules in the scattering theory forces the remarkable property of the total S-matrix: it is factorized in the standard manner into two-particle S-matrices [1, 2, 8]. Any multiparticle S-matrix element can be expressed in terms of the two-particle amplitudes, the unitarity and analyticity of the total S-matrix being the consequence of the same properties of the two-particle one. Furthermore, the factorized form of the S-matrix requires special functional relations (the so-called factorization equations) for the two-particle amplitudes to be satisfied. In a number of interesting cases one can work out the two-particle S-matrix explicitly by solving the factorization equations together with the unitarity and analyticity conditions. In this way the factorized S-matrices having $O(N)$ [3],

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