Commun. Math. Phys. 69, 57–80 (1979)

Communications in
Mathematical
Physics

Computation of Quantum Fluctuations Around Multi-Instanton Fields from Exact Green's Functions: The CP^{n-1} Case

B. Berg[1] and M. Lüscher[2]

[1] II. Institute für Theoretische Physik, D-Hamburg, Federal Republic of Germany
[2] Deutsches Elektronen-Synchrotron DESY, D-Hamburg, Federal Republic of Germany

Abstract. We calculate exactly the contribution of instanton fields to the partition function of $\mathbb{CP}^{n-1}$ models in two dimensions. For $n=2$, the pure instanton gas is infrared finite, infinitely dense and generates a mass dynamically. For $n \geqq 3$, the gas corresponds to a system with complicated n-body interactions, whose properties are yet to be explored.

1. Introduction

Previous work [1] based on the $1/n$ expansion of $\mathbb{CP}^{n-1}$ models [2] in two dimensions revealed that topologically non-trivial fields make a significant contribution to the low energy dynamics of the fundamental particles in these theories. How much of these effects is due to instantons was not clear, however, because their contribution appeared to be infrared divergent. In fact, the single instanton contribution is proportional to

$$\int_0^\infty d\lambda \lambda^{n-3}; \quad \lambda: \text{scale size of the instanton,} \tag{1}$$

which diverges for large λ. Assuming the instanton gas is dilute does not help but merely exponentiates the divergence. In other words, the thermodynamic limit of the dilute instanton gas does not exist.

Equation (1) says that large instantons are more probable than small ones so that the instanton gas may well be dense, i.e. the average thickness of an instanton might be much larger than the mean separation between instantons. Once the diluteness assumption is dropped, it may turn out that the exact instanton gas has a thermodynamic limit. In particular, for the partition function Z of the instanton gas in a volume V this would mean that

$$\ln Z = p.V \quad (V \to \infty) \tag{2}$$

where p is proportional to the pressure of the gas. To investigate the question of

0010-3616/79/0057/$04.80