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The Third Law of Black Hole Mechanics: A Counterexample

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Abstract. The collapse of a spherically symmetric charged thin shell in a Reissner-Nordstrøm field can lead to an extreme black hole. No contradiction to the assumption of Cosmic Censorship results.

1. Introduction

The third law of black hole mechanics is the conjecture, first formulated by Bardeen et al. [1], that reads: "It is impossible by any procedure, no matter how idealized, to reduce $\varkappa$ to zero by a finite sequence of operations."

Here

$$\varkappa = \frac{\sqrt{m^2 - a^2 - e^2}}{2m^2 - e^2 + 2m\sqrt{m^2 - a^2 - e^2}}$$

is the so-called surface gravity of the black hole of total mass m, specific angular momentum a and total charge e. Zero $\varkappa$ means that the hole is extreme, i.e.,

$$m^2 = a^2 + e^2 .$$

We devise a counterexample to this conjecture: a process that is capable of producing an extreme black hole in a finite interval of advanced time. The process can be described as follows. Consider asymptotic observers throwing a thin shell of charged incoherent matter towards a Reissner-Nordstrøm black hole in a spherically symmetric way. In the past of the shell, the values of the mass and charge parameters of the spacetime are m_1 and e_1, in its future, they are m_2 and e_2. The shell starts with total *injection energy* E (energy with respect to the asymptotic observers) and total charge e, which satisfy the conservation laws:

$$e_2 = e_1 + e, \qquad m_2 = m_1 + E$$

(see Sect. 2). The motion of such shells has been thoroughly studied [2–5]. One particular result is that the shell implodes unless a minimal, or *bounce radius*, R_b, is

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