Commun. math. Phys. 68, 245—258 (1979)

Communications in Mathematical Physics

Borel Summability of the Mass and the S-Matrix in φ^4 Models

J.-P. Eckmann[1] and H. Epstein[2]

[1] Department of Theoretical Physics, University of Geneva, CH-1211 Geneva, Switzerland, and
[2] Institut des Hautes Etudes Scientifiques, F-91440 Bures-sur-Yvette, France

Abstract. We show that, in the φ_2^4 theory, the physical mass and the two-body S-matrix are Borel summable in the coupling constant λ at $\lambda=0$.

1. Introduction

In this paper we show that in the φ_2^4 theory the following objects are Borel summable in the coupling constant at zero: (i) the momentum space analytic functions for every complex momentum in an open set containing the Euclidean points; (ii) the physical mass and the field strength renormalization constant; (iii) the two-body S-matrix in the elastic region. The proofs of (i) and (ii) have been written so that they extend straightforwardly to the case of φ_3^4 with the help of the cluster expansion as given by Magnen and Sénéor [14] and Burnap's work [3]. By contrast, the proof of (iii) uses the analyticity in the coupling constant of the irreducible kernels, known for the even φ_2^4 theory from Spencer's analysis [16]. It could be extended to non-even φ_2^4 theories by using the work of Koch [13]. The method extends to the massive Sine-Gordon model [9], where it yields analyticity in the coupling constant around 0. The principle of the method is clearly present in [9].

The Schwinger functions of the φ_2^4 theory are given by

$$S_n(x_1, \ldots, x_n, \lambda, \zeta)$$
$$= \lim_{\Lambda \uparrow \mathbb{R}^2} N^{-1}(\Lambda, \lambda, \zeta) \int d\mu_\zeta(\varphi)\varphi(x_1)\ldots\varphi(x_n) \exp\left| -\lambda \int_\Lambda :\varphi^4(x): d^2x \right|, \tag{1}$$

where $d\mu_\zeta$ is the Gaussian measure with (bare) mass $\zeta^{1/2}$ and : : denotes Wick ordering with the same mass. N is the obvious normalization factor. For $\lambda \geqq 0$ sufficiently small and $\zeta > 0$ sufficiently large the theory is known to exist [11]. Its physical mass will be denoted $m(\lambda, \zeta)$, and the first threshold above it, $2m'(\lambda, \zeta)$. The natural scaling law

$$S_n(\varrho x_1, \ldots, \varrho x_n, \varrho^{-2}\lambda, \varrho^{-2}\zeta) = S_n(x_1, \ldots, x_n, \lambda, \zeta) \tag{2}$$

0010-3616/79/0068/0245/$02.80