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Lorentz Covariance and Kinetic Charge

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Abstract. There is a one-to-one correspondence between inequivalent covariant displaced Fock representations of the free relativistic field and the 1-cohomology of the Poincaré group with coefficients in the 1-particle space.

Representations with positive energy are obtained from cocycles with finite energy which have particle-like properties and are interpreted as condensed states of matter without a sharply defined mass.

The 1-cohomology groups of $\mathscr{P}_+^\uparrow$ are calculated. These are trivial in 3- or 4-dimensional space-time, or if the mass is non-zero. Non-trivial cocycles for subgroups lead to representations in which $\mathscr{P}$-invariance is spontaneously broken. We recover $\mathscr{P}$-invariance in a direct integral representation possessing a gauge group, and a superselection structure labelled by the velocities of the condensed states of matter which are the cocycles determining each irreducible component of the representation. A model in 4-dimensional space-time is constructed.

1. Cohomological Classification of Displaced Fock Representations

Let U be a representation of the Poincaré group $\mathscr{P}_+^\uparrow$ acting on a Hilbert space $\mathscr{H}$ and satisfying the spectral condition:

$$P^0 \geqq 0 \; (P^0)^2 \geqq \mathbf{P}^2 ,$$

where P^μ is the self-adjoint generator of space-time translations in the direction x^μ, $\mu = 0, 1, \ldots, s$. If m is the mass of U, then $\mathscr{H}$ may be realised as a class of solutions of a manifestly covariant family of wave equations and subsidiary conditions:

$$(\Box + m^2)\varphi_\alpha(\mathbf{x}, t) = 0 \quad D^{\alpha\beta}\varphi_\beta(\mathbf{x}, t) = 0 ,$$
$$\alpha = 1, 2, \ldots ,$$

where D is a tensor of differential operators chosen to remove unwanted spin components.

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