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Quasilinear Hyperbolic Systems

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Abstract. We construct global solutions for quasilinear hyperbolic systems and study their asymptotic behaviors. The systems include models of gas flows in a variable area duct and flows with a moving source. Our analysis is based on a numerical scheme which generalizes the Glimm scheme for hyperbolic conservation laws.

We consider the initial value problem for quasilinear partial differential equations of the following form

$$\frac{\partial u}{\partial t}+\frac{\partial f(u)}{\partial x}=g(x,u), \quad -\infty<x<\infty, \quad t\geqq 0, \tag{0.1}$$

$$u(x,0)=u_0(x), \quad -\infty<x<\infty. \tag{0.2}$$

Here $u=u(x,t)$ is an n-vector, f is a smooth n-vector-valued function of u, and g and $\frac{\partial g}{\partial u}$ are piecewise continuous n-vector-valued function of x, and are continuous in u. System (0.1) is assumed to be strictly hyperbolic, that is $\partial f(u)/\partial u$ has real and distinct eigenvalues $\lambda_1(u)<\lambda_2(u)<\ldots<\lambda_n(u)$ for each u. In general (0.1) and (0.2) do not possess smooth solutions, and we look for weak solutions, that is, solutions satisfying

$$\iint_{t\geqq 0}\left(u\frac{\partial\varphi}{\partial t}+f(u)\frac{\partial\varphi}{\partial x}-g(x,u)\varphi\right)dx\,dt+\int_{-\infty}^{\infty}u_0(x)\varphi(x,0)\,dx=0 \tag{0.3}$$

for any smooth function $\varphi(x,t)$ with compact support in $t\geqq 0$. The purpose of this paper is to construct solutions for (0.1) and (0.2) and study their asymptotic behavior as the time variable t tends to infinity.

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