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Link Between the Geometrical and the Spectral Transformation Approaches in Scattering Theory

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Abstract. We show how the Enss's geometrical proof of asymptotic completeness may be set on commutator properties.

In [1], the author introduces a new method to prove the asymptotic completeness for potential scattering. This method is based on geometrical concepts which are very intuitive and linked to the classical free evolution. Practically the author introduces a decomposition of the identity on $L^2(\mathbf{R}^n): \mathbf{1} = P_{\text{out}} + P_{\text{in}}$ such that $(\Omega_{\mp} - \mathbf{1})P_{\substack{\text{out}\\ \text{in}}}$ are compact and that enables him to prove asymptotic completeness.

We show in this article how to introduce a similar decomposition of the identity $\mathbf{1} = P^+ + P^-$, from the commutator relation $i[H_0, A] = H_0$, where A is the well known dilatation group generator on $L^2(\mathbf{R}^n)$ (see [2]), and P^+ and P^- are the spectral projectors of A respectively on the positive and negative parts of the A spectrum.

To prove Lemma 1, we use commutator properties to get interesting differential inequalities, as it has been used in [3] and partially refined in [4], in order to obtain hamiltonian resolvent estimations in the neighbourhood of the continuous spectrum.

Definitions

Let Δ be the laplacian on $L^2(\mathbf{R}^n)$, $H_0 = -\Delta$ and A be the generator of the dilatation group on $L^2(\mathbf{R}^n)$ normalized in such a way that

$$e^{+iA\alpha} H_0 e^{-iA\alpha} = H_0 e^{-\alpha}.$$

Let us note P^+ and P^- the spectral projectors of A on the positive and negative parts of its spectrum.

Lemma 1. *Let g be a regular function with compact support in $\mathbf{R}^+ - \{0\}$ and $0 \leqq \mu' < \mu$.*

Then there is a constant c (depending on g, μ, μ') such that

$$\left\| \chi^{\pm}(t) \frac{1}{|A+i|^{\mu}} e^{-iH_0 t} g(H_0) P^{\pm} \right\| \leqq c \frac{1}{|t|^{\mu'}}.$$

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