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Small Perturbations of C^*-Dynamical Systems

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Abstract. It is shown that if δ is the generator of a strongly continuous one-parameter group of *-automorphisms of a C^*-algebra A and δ' is an unbounded *-derivation of A with the same domain as δ, then $\delta+\alpha\delta'$ is also a generator for all sufficiently small real numbers α.

The perturbation theory of strongly continuous one-parameter contraction semi-groups $\{e^{tT}: t \geqq 0\}$ on Banach spaces shows that several features of these systems are stable under relatively bounded perturbations [6, 8]. For example if T' is a dissipative operator with the same domain $\mathscr{D}$ as T, then $T+T'$ is the generator of some contraction semi-group, provided that

$$\|T'x\| \leqq \alpha\|x\| + \beta\|Tx\|$$

for all x in $\mathscr{D}$, for some constants α and $\beta<1$.

In the C^*-algebraic model of a quantum dynamical system, the time evolution is represented by a strongly continuous one-parameter group of *-automorphisms $\{e^{t\delta}: t\in\mathbb{R}\}$ of a C^*-algebra A, where the generator δ is a closed unbounded *-derivation. Longo [7] has shown that in this case, any *-derivation δ' with the same domain is automatically relatively bounded with respect to δ. In this note it will be shown that δ' is also necessarily dissipative, and therefore $\delta+\alpha\delta'$ is a generator for all sufficiently small α (cf. [4, Sect. 5]).

Longo's result also applies if δ is any closed *-derivation (not necessarily a generator), and he asked whether δ' is then necessarily closable. For commutative C^*-algebras, an affirmative answer to this problem was given in [3, Theorem 5.3]. The proof there involved showing that any (maximal) closed ideal containing a and $\delta(a)$ also contains $\delta'(a)$. Since the maximal ideals in a commutative C^*-algebra are of codimension 1 and have zero intersection, this enabled a very specific description of δ' to be given in terms of δ. For non-commutative C^*-algebras, it will be shown here that $\delta'(a)$ again belongs to the closed ideal generated by a and $\delta(a)$, and a partial answer to Longo's question will be given. All the results of this

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