Commun. Math. Phys. 68, 9–27 (1979)

Communications in Mathematical Physics

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Decay of Correlations in Classical Lattice Models at High Temperature*

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Abstract. In classical statistical mechanical lattice models with many body potentials of finite or infinite range and arbitrary spin it is shown that the truncated pair correlation function decays in the same weighted summability sense as the potential, at high temperature.

1. Introduction

We consider a lattice model on Z^m with single spin space X and interaction φ. Here φ denotes a real valued function $\cup_A X^A$, the union running over all nonempty finite subsets, A, of the lattice Z^m. Under suitable conditions on φ, and for sufficiently small β, the interaction $\beta\varphi$ has a unique Gibbs state σ. We shall study the decay of the truncated two point function for σ. More precisely, and somewhat more generally, we study the behavior of $\psi(a) \equiv \sigma(fg_a) - \sigma(f)\sigma(g_a)$ where f and g are bounded real functions on $X^{(Z^m)}$ and g_a is the translate of g by an element a in Z^m.

For what potentials φ does $\psi(a)$ decay in some specified manner as $a \to \infty$ (e.g., exponentially)? This question has been addressed in numerous works, in which, under a variety of different conditions on the form of φ, an assumed decay rate for φ at large distances is shown to imply some related kind of decay rate for $\psi(a)$ as $a \to \infty$. The restrictions on the form of φ usually involve a restriction on either the range of φ (e.g., finite range), the many bodiedness of φ (e.g., pair interactions), or on the cardinality of X (e.g., cardinality two), or some combination of these, and sometimes further form restrictions designed to allow use of transfer matrix methods or correlation inequalities. For a survey of the extensive literature up to 1974 we refer the reader to Duneau, Souillard and Iagolnitzer [4], and to Ruelle [16]. We also refer the reader to Ruelle [17] for background structure. We mention here only the recent papers of Israel [11], Sylvester [18], and Holley and Stroock [8], which obtain exponential decay for some interactions of what may be called locally finite range. For other recent work see also [20–22].

* Research partially supported by the National Science Foundation under Grant MCS 78-00680

0010-3616/79/0068/0009/$03.80