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On the Structure of Symmetry Generators

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Abstract. In a field theoretic framework we investigate generators of symmetry transformations induced by conserved local, not necessarily translationally covariant currents. Assuming the invariance of the vacuum and a mass gap, it is shown that the generator on one-particle states in general can be any polynomial of the generators of the Poincaré group and the internal symmetries. We give an example showing that the generator, defined as an integral over a conserved current, in spite of leaving the vacuum invariant, need not be self-adjoint.

1. Introduction

We consider a symmetry of a Wightman field theory induced by a conserved local current $j_\mu(x)$ which, in the general case, is not translationally covariant. We assume that $j_\mu(x)$ commutes with the basic fields for space-like separations and hence currents inducing supersymmetries are not considered here. The corresponding symmetry generator Q has been investigated in [1], [2] and papers quoted therein. In particular, it has been shown (under the assumption of a mass gap and the invariance of the vacuum) that Q extends to an operator on asymptotic scattering states. Its action on asymptotic fields is given by

$$i[Q(x+y), \psi_\kappa(x)] = \sum_\lambda \check{P}_{\kappa\lambda}(y, \partial_x)\psi_\lambda(x). \tag{1.1}$$

Here $Q(\xi)$ denotes the generator Q translated by $\xi \in \mathbb{R}^4$, $\check{P}_{\kappa\lambda}(y, \partial_x) = \check{\Xi}_{\kappa\lambda}(y, \partial_x) + \check{\Lambda}_{\kappa\lambda}(y, \partial_x)\partial_{x_0}$. $\check{\Xi}$, $\check{\Lambda}$ are polynomials in $y \in \mathbb{R}^4$ and spatial derivatives ∂, vanishing if the fields ψ_κ, ψ_λ have different mass. $\{\psi_\kappa\}$ is a (countable) complete set (cyclic with respect to the vacuum Ω) of linearly independent asymptotic incoming fields (We could also have chosen outgoing fields: Q does not depend on the choice in or out).

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