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Shift Automorphisms in the Hénon Mapping

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Abstract. We investigate the global behavior of the quadratic diffeomorphism of the plane given by $H(x, y) = (1 + y - Ax^2, Bx)$. Numerical work by Hénon, Curry, and Feit indicate that, for certain values of the parameters, this mapping admits a "strange attractor". Here we show that, for A small enough, all points in the plane eventually move to infinity under iteration of H. On the other hand, when A is large enough, the nonwandering set of H is topologically conjugate to the shift automorphism on two symbols.

Several numerical studies have recently appeared [3, 4, 7, 8] on the dynamics of the diffeomorphisms of the plane

$$H(X, Y) = (1 + Y - AX^2, BX) .$$

Interest in these maps [12, 14, 5] has been prompted by Hénon's numerical evidence [8] for a "strange attractor" when $A = 1.4$, $B = 0.3$. Feit [4] has shown, for $A > 0$ and $0 < B < 1$, that the non-wandering set $\Omega(H)$ is contained in a compact set, and that all points outside this set escape to infinity. Curry [3] has shown that, for Hénon's values of the parameters, one of the fixed points has a topologically transverse homoclinic orbit, and hence that there is a horseshoe embedded in the dynamics of the map.

The present note is intended to clarify the behavior of the mapping H for parameter values far from those where "strange attractors" have been observed. Hénon and Feit have noted that for $B = 0.3$ and A outside a certain interval (roughly $[-0.12, 2.67]$) no attractors are observed; numerically, all points seem to escape to infinity. We exhibit, for any $B \neq 0$, a pair of A values, $A_0 < 0 < A_2$, such that the non-wandering set $\Omega(H)$ is empty for $A < A_0$, but for $A > A_2$, $\Omega(H)$ is the zero-dimensional basic set obtained from Smale's horseshoe construction [9, 11, 13]. We begin by rewriting the map in a more convenient form; then we establish Feit's result (for all $A, B \neq 0$) in a version more suited to our purposes, by

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