Commun. Math. Phys. 67, 109–119 (1979)

Communications in Mathematical Physics

On the Decoupling of Massive Particles in Field Theory

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Abstract. The article examines the Feynman amplitude when some of the mass parameters are scaled to infinity. Contributions from diagrams containing the scaled mass go to zero provided no particles are massless and BPHZ subtractions are used.

I. Introduction

During the last few years there have been numerous applications of the decoupling theorem [1], [2]. These applications have made desirable a more stringent proof of the theorem. It will be shown that the heavy sector decouples perturbatively when BPH subtractions are used, in the case of a theory with two mass scales and momenta in the Euclidean regime.

As in Weinberg's power counting theorem the problems are mainly technical. The works of Appelquist [3], Anikin, Polivanov, and Zavialov [4], Bergere and Zuber [5], and Bergère and Lam [6] have shown that the α-parametric integral representation allows one to write down a closed expression for the renormalized Feynmann amplitude.

The rest of the paper is organized in the following way:

Section II gives the definition of the Feynman amplitude in the case of a scalar theory. In Sect. III a simple proof of the decoupling theorem is outlined for a scalar theory where technical problems are minimalized. The generalization to theories with spin and derivative couplings can be found in Sect. IV. Section V contains a short discussion of the results. Two appendices are devoted to technical questions.

II. Parametric Integral Representation of the Feynman Amplitude

To any connected Feynman graph G with L lines and V vertices corresponds the Feynman amplitude (in Euclidian space)

$$\tilde{F}(p) = \int \prod_{l \in L} \frac{d^4 k_l}{(2\pi)^4 (k_l^2 + m_l^2)} \cdot \prod_{v \in V} (2\pi)^4 \delta^{(4)} \left(p_v - \sum_{l \in L} \langle v, l \rangle \, k_l \right)$$

where p_v denotes the sum of the external momenta beginning at vertex v and $\langle v, l \rangle$ the incidence matrix on $V \times L$.

0010-3616/79/0067/0109/$02.20