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On the Cook-Kuroda Criterion in Scattering Theory*

Tosio Kato

Department of Mathematics, University of California, Berkeley, California 94720, USA

Abstract. A new criterion of the Cook-Kuroda type for the existence of the wave operator in the two-space scattering theory is introduced. The condition is quite simple, but it generalizes not only the original Cook-Kuroda condition but also its generalization recently given by Schechter. Specialized to the one-space case, it is actually equivalent to Schechter's condition for an optimal choice of factorization. An application to potential scattering leads to a new result.

1. Introduction

Recently Schechter [1] and Simon [2] generalized the 20-year-old Cook-Kuroda criterion [3, 4] for the existence of the wave operator in scattering theory. The purpose of the present paper is to contribute another generalization in the context of *two-space scattering theory* [5]. Our condition (Theorem I) has several advantages. First, it is formally simpler than others [1–4], involving only bounded operators. Second, it has a simple, purely time-dependent proof. Third, it is valid in the two-space setting without any extra assumptions on the identification operator J except that J is bounded. Fourth, Schechter's theorem can easily be reduced to ours, so that our results contain a simplified proof of a two-space version of his theorem. At the same time, this shows that our result is in general stronger than Schechter's.

On the other hand, Schechter's condition is extremely flexible, involving a (formal) factorization of the perturbation that can be chosen in many different ways. In fact we shall show that some favorable choices of the factorization lead to a result equivalent to ours (Theorem III).

Let us first state our theorems. In two-space scattering theory, one considers two selfadjoint operators H_j, $j=1,2$, each acting in its Hilbert space $\mathfrak{H}_j$, and a bounded linear operator J (the *identification operator*) on $\mathfrak{H}_1$ to $\mathfrak{H}_2$. We denote by $U_j(t)=\exp(-itH_j)$ the unitary group generated by $-iH_j$. The associated *wave*

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