Commun. Math. Phys. 67, 17–42 (1979)

Communications in
Mathematical
Physics

Existence of Long-Range Order in the Migdal Recursion Equations

P. M. Bleher[1] and E. Žalys[2]

[1] Institute of Applied Mathematics, Academy of Sciences, Moscow, USSR
[2] Institute of Mathematics and Cybernetics, Academy of Sciences of Lithuania SSR, SU-232600 Vilnius, USSR

Abstract. A modification of XY – model is introduced for which Migdal recursion equation are exact. High- and low-temperature fixed points of these equations are investigated. As a result the existence of long-range order at low temperature and its absence at high temperature are proved rigorously for the model under consideration in the case when dimension $d>2$.

1. Introduction

The renormalization group method is widely used in physical works on phase transitions (see for instance [1–6]). At the same time a rigorous application of the method has been possible only for a narrow class of models (see [7–11]). But the simplicity and generality of the main ideas underlying the renormalization group provide a strong motivation for using it as a tool for rigorous investigations.

In this paper we shall use the renormalization group method to prove the existence of long-range order at low temperatures in a modified d-dimensional XY-model, when $d>2$. This model was first discussed in the papers by Migdal ([5, 6]) and was defined for all real values of the dimension d in the following way.

Let at first $d \geqq 2$ and $v \geqq 2$ be integers. We shall consider the hierarchical sequence of cubes $V_n, n=0,1,\dots$ in d-dimensional Euclidean space $\mathbb{R}^d$:

$$V_n = \{x=(x_1,\dots,x_d)\in\mathbb{R}^d,\ 0\leqq x_i \leqq v^n;\ i=1,\dots,d\}\ .$$

For every n the whole space $\mathbb{R}^d$ is covered by cubes V_{n,a_m} which appear as shifts of the V_n by integer vectors $a_m = mv^n$, $m\in\mathbb{Z}^d$ proportional to v^n. Clearly, two cubes V_{n,a_m}, $V_{n,a_{m'}}$, $m \neq m'$ either do not intersect or have a whole face in common.

Let us denote by S the set of closed $(d-1)$-dimensional elementary faces on the lattice $\mathbb{Z}^d$. Each face $s\in S$ is defined by an integer vector $m\in\mathbb{Z}^d$ (the point on the face with least coordinates assuming that the lexicographical ordering is introduced) and by the number i of the axis to which the face s in orthogonal. Thus each face can be written as $s_{m,j}$ where we denote the configuration space of the functions

0010-3616/79/0067/0085/$01.20