

## Classification of Gravitational Instanton Symmetries

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**Abstract.** We classify the action of one parameter isometry groups of Gravitational Instantons, complete non singular positive definite solutions of the Einstein equations with or without  $\Lambda$  term. The fixed points of the action are of 2-types, isolated points which we call “nuts” and 2-surfaces which we call “bolts”. We describe all known gravitational instantons and relate the numbers and types of the nuts and bolts occurring in them to their topological invariants. We perform a 3+1 decomposition of the field equations with respect to orbits of the isometry group and exhibit a certain duality between “electric” and “magnetic” aspects of gravity. We also obtain a formula for the gravitational action of the instantons in terms of the areas of the bolts and certain nut charges and potentials that we define. This formula can be interpreted thermodynamically in several ways.

### 1. Introduction

There has been considerable interest recently in “Instantons” in Yang-Mills Theory [1–3]. They may be defined as non singular solutions of the classical equations in 4-dimensional Euclidean space. They provide stationary phase points in the path integral for the amplitude to tunnel between two topologically distinct vacua [4, 5] and they may play a role in confinement. Instantons also contribute to the anomalous divergence of the axial vector current [2] and they may lead to the decay of baryons into leptons. Because gravity and supergravity are gauge theories like Yang-Mills it seems reasonable to suppose that gravitational instantons may play a similar important role. We shall define a gravitational instanton to be a non singular complete positive definite metric which satisfies the classical vacuum Einstein equations or the Einstein equations with a  $\Lambda$  term. The  $\Lambda$  term can be regarded as a Lagrange multiplier for the 4-volume  $V$  or it may arise from the Lagrangians of certain supergravity theories [6, 7]. One class of gravitational instantons that has been extensively studied already is the Kerr-Newman family of metrics [7–12]. In these solutions one can remove the apparent