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# Graded Manifold Theory as the Geometry of Supersymmetry*

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**Abstract.** Building upon Kostant's graded manifold theory, we present a new way of introducing spinors into the spacetime manifold, by expanding the algebra of functions on spacetime to a graded algebra. The elements of differential geometry are generalized to accomodate the expanded algebra of functions and in this enriched geometry we find the elements of supersymmetry and of supergravity theory. The geometrical role of the supergravity fields is discussed and a derivation of their transformation rules is given.

## I. Introduction

Symmetry transformations mixing particles of different spin were introduced several years ago under the name of supersymmetry [1, 2]. In the simplest case, due to Wess and Zumino [2], a scalar field $\phi$ and an anticommuting spinor field $\chi$ are given the following variations

$$\delta\phi=\bar{\varepsilon}\chi \tag{1.1a}$$

$$\delta\chi=(\partial\!\!\!/\phi)\varepsilon\,, \tag{1.1b}$$

where $\varepsilon$ is taken to be a constant anticommuting spinor. The action integral for a free scalar field and a free spinor field is easily seen to be invariant under these variations [2].

A characteristic feature of supersymmetry transformations is that the symmetrized product of two supersymmetry transformations is always an infinitesimal translation. One finds that,

$$(\delta_1\delta_2+\delta_2\delta_1)\phi,\chi=2\bar{\varepsilon}_1\gamma^\mu\varepsilon_2\partial_\mu\phi,\chi\,. \tag{1.2}$$

Thus supersymmetry transformations are in some sense an extension of the algebra of infinitesimal spacetime translations. This suggested that a theory of local supersymmetry, where the parameters $\varepsilon$ are taken to be functions of

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