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Critical Point Inequalities and Scaling Limits

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Abstract. A refined and extended version of the Buckingham-Gunton inequality relating various pairs of critical exponents is shown to be valid for a large class of statistical mechanical models. If this inequality is an equality (in the refined sense) and one of the critical exponents has a non-Gaussian value, then any scaling limit must be non-Gaussian. This result clarifies the relationship between the nontriviality or triviality of the scaling limit for ordinary critical points in four dimensions (or tricritical points in three dimensions) and the existence of logarithmic factors in the asymptotics which define the two critical exponents.

I. General Results

In this paper, we use some surprisingly simple probabilistic arguments to obtain rather detailed information about the critical behavior of a fairly large class of statistical mechanical systems. For example, in the case of a four dimensional Ising model we relate (see Corollary 2.7 below) the possible existence of logarithmic factors in the large distance behavior of the two point correlation (at the critical point) and in the small external field behavior of the free energy (at the critical temperature) to the question of the possible triviality (i.e., Gaussian-ness) of the Kadanoff block spin scaling limit [14, 22, 21]. One view of this result is that it provides sufficient conditions for proving that the scaling limit is nontrivial and thus perhaps also for proving that nontrivial ϕ^4 field theories exist in four dimensional spacetime [2, 15, 31]. This point of view underlies many of the critical exponent results of Glimm and Jaffe (see [16] and the references listed there). Another view is that it merely yields a new critical exponent inequality relating the two possible logarithmic factors – an inequality which must be strict if the scaling limit is to be Gaussian (as it "should" be). This latter view is consistent with various calculations which imply such a strict inequality (see the discussion following Corollary 2.8 below and the references cited there).

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