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Phase Transitions in Ferromagnetic Spin Systems at Low Temperatures

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Abstract. We consider the problem of the existence of first order phase transitions in ferromagnetic spin systems at low temperatures. A criterion is given for the existence of phase transitions in terms of an algebraic system canonically associated with any interaction. The criterion involves finding out if the greatest common divisor of few polynomials belongs to the ideal generated by these polynomials.

In connection with results published earlier, this work yields a description of all translation invariant (also of periodic and quasi-periodic) equilibrium states at low temperatures.

Introduction

A system exhibits first order phase transition if it has more than one invariant equilibrium state. One would like to be able to say if for given interaction (and external parameters) there is a phase transition, how many pure phases are there, and how to distinguish the phases. Furthermore, one would like to test the extremal invariant states against the breakdown of various symmetries and to discuss their clustering properties.

A solution of these problems for ferromagnetic spin $\frac{1}{2}$ systems at low temperatures on $\mathbb{Z}^{\nu}$ was described in our note [15] which can serve as an introduction to the present work. In Sect. 5 we prove a strengthened version of the conjecture of [15]. Extension of these results to higher spin systems is contained in [32].

The problem of phase transitions at low temperatures has received considerable attention. The case of Ising model and its perturbations was treated in the sixties by Griffiths [9] and Dobrushin [3]. This was generalized in [1, 8]; other relevant papers can be traced through [10, 11, 13].

More recently a theory of phase transitions for any finite number of ground states has been developed, first in case when the ground states are related by symmetries of the interaction [5] and then more complete theory in [25, 26]. In a number of cases existence of phase transitions has been proved with help of the Reflection Positivity method [6]. And ferromagnetic spin $\frac{1}{2}$ systems have been treated in [12, 13, 20–23].

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