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Time Evolution of Gibbs States for an Anharmonic Lattice

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Abstract. In this paper we study the time evolution of a regular class of states of an infinite classical system of anharmonic oscillators. The conditional probabilities are investigated and an explicit form for these is given.

1. Introduction

One of the main problem in non equilibrium Statistical Mechanics is to study the time evolution of states (i.e. probability measures on the phase space) of infinite interacting classical systems. A natural way is to consider the time evolution as described by a flow on the phase space arising from the Newton law of the motion.

The problem of constructing such a flow was solved in a satisfactory way for some classes of particle systems in [1], [2] and for anharmonic oscillators in [3]. Other results which are specifically related to the equilibrium situation were obtained in [4–8].

The next step is to study the time evolution of states, implemented by the flow on the phase space. An approach proposed in [9] and [10] is based on the hypothesis that a class of physically interesting states, the Gibbs states with respect to some Hamiltonian is preserved in the course of the evolution (the equilibrium states are precisely those states which are Gibbs with respect to the Hamiltonian governing the motion). The main advantage of this approach is that the change in time of the Hamiltonian of a given Gibbs state is described in a simple way, directly referred to finite-volume dynamics.

Such an approach was studied in [9] in the case of one dimensional hard core system interacting via a two body, bounded, short range potential. One of the main points in [9] is the use of the cluster dynamics that, roughly speaking, says that such systems behave in time as if they were formed by non interacting groups consisting of a finite number of particles.

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