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Micro-Analyticity of the *S*-Matrix and Related Functions

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Abstract. It is shown that the singularity spectrum of the phase space integral associated with any partially ordered Landau diagram is confined to a variety defined by a set of modified Landau equations. These equations are similar to the ordinary Landau equations but involve limiting procedures. The variety defined by the modified equations coincides with the variety defined by the ordinary equations except at points called $u=0$ points. Next the causal parts of the sets defined by the modified Landau equations are defined, in a natural way, and it is conjectured that the singularity spectrum of the *S*-matrix is confined to the union of the causal parts of the singularity spectra of the phase space integrals. An analogous conjecture on general bubble diagram functions asserts that the singularity spectrum of each of these functions is confined to sets defined by the modified Landau equations augmented by appropriate positive-α and negative-α conditions. Generalized Landau equations are introduced. These equations do not involve limiting procedures, but provide a useful partial characterization of the sets defined by the modified Landau equations augmented by these positive-α and negative-α conditions.

§0. Introduction

The primary purpose of this paper is to formulate a conjecture on the singularity spectrum of the *S*-matrix. This conjecture is designed to be compatible both with unitarity and with the macro-causality requirement that momentum-energy can be transferred over macroscopic distances only by stable particles. These requirements are severe, and our conjecture appears to provide a satisfactory point of departure for the analysis of the singularity structure of the *S*-matrix within the framework of the theory of holonomic functions (=functions satisfying a holonomic (=maximally overdetermined) system of (micro-)differential equations)

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