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The Classical Field Limit of Scattering Theory for Non-Relativistic Many-Boson Systems. I

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Abstract. We study the classical field limit of non-relativistic many-boson theories in space dimension $n \geqq 3$. When $\hbar \to 0$, the correlation functions, which are the averages of products of bounded functions of field operators at different times taken in suitable states, converge to the corresponding functions of the appropriate solutions of the classical field equation, and the quantum fluctuations are described by the equation obtained by linearizing the field equation around the classical solution. These properties were proved by Hepp [6] for suitably regular potentials and in finite time intervals. Using a general theory of existence of global solutions and a general scattering theory for the classical equation, we extend these results in two directions: (1) we consider more singular potentials, (2) more important, we prove that for dispersive classical solutions, the $\hbar \to 0$ limit is uniform in time in an appropriate representation of the field operators. As a consequence we obtain the convergence of suitable matrix elements of the wave operators and, if asymptotic completeness holds, of the S-matrix.

1. Introduction and Statement of the Problem

Since the early days of quantum mechanics it has been a natural question to compare the classical and quantum mechanical descriptions of physical systems. One of the oldest and by now best known relations between the two theories goes back to Ehrenfest [1]. Only recently however was this relation put on a firm mathematical basis by Hepp [6] who proved that in the limit $\hbar \to 0$ the matrix elements of bounded functions of quantum observables between suitable $\hbar$-dependent coherent states tend to classical values evolving according to the appropriate classical equation. Furthermore he proved that the quantum mechanical fluctuations evolve according to the equation obtained by linearizing the quantum mechanical evolution equation around the classical solution. However his analysis is limited to finite time intervals and therefore does not

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