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Non-Translation Invariant Gibbs States with Coexisting Phases

II. Cluster Properties and Surface Tension*

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Abstract. We prove cluster properties of the spatially inhomogeneous Gibbs states in symmetric two component lattice systems obtained at large (equal) values of the fugacity. We also prove that the surface tension of these systems is given by an integral over the density variation in this state; Gibbs' formula. An alternative formula for the surface tension is also derived.

1. Introduction

In a previous paper [1] we proved for a class of two-component $A-B$ lattice gas systems in three dimensions the existence of Gibbs states in which there is a spatial segregation into an A-rich and a B-rich phase with a "sharp interface". These states are obtained, at high values of the chemical potential μ, $\mu=\mu_A=\mu_B$, by taking the infinite volume limit of a system with boundary conditions favoring $A(B)$ particles in the upper (lower) part of a box. This is entirely analogous to the existence of such nontranslation invariant states for ferromagnetic Ising spin systems at sufficiently low temperatures in three or more dimensions. The latter was first proven by Dobrushin [2] whose methods we used heavily in [1].

The purpose of this paper is to prove further properties of this nontranslation invariant Gibbs state: extremality, exponential clustering of correlation functions (no long range transverse part), and asymptotic behavior far from the interface. We also prove that the surface tension in these states is given by an integral over the correlation functions. This justifies a commonly used expression due originally to Gibbs. The methods used here are, like in [1], based on the work of Dobrushin [3]. For this reason we generally omit details of the proofs. Our results about the surface tension are new and apply also to the Ising model.

We use notation, definitions and results of [1] and we treat only the following model: we have two kinds of particles A and B with chemical potentials $\mu_A=\mu_B=\mu$. There is at most one particle at each point of $\mathbb{Z}^3$ and the presence of a

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