Commun. math. Phys. 65, 295–312 (1979)

Communications in Mathematical Physics

On the Ergodic Properties of Nowhere Dispersing Billiards

L. A. Bunimovich

P. P. Shirshov Institute of Oceanology, Academy of Sciences USSR, Moscow 117218, USSR

Abstract. For billiards in two dimensional domains with boundaries containing only focusing and neutral regular components and satisfacting some geometrical conditions *B*-property is proved. Some examples of three and more dimensional domains with billiards obeying this property are also considered.

Introduction

In the present paper dynamical systems with elastic reflections, usually called billiards, are studied. From the point of the general theory of dynamical systems a billiard can be considered as a geodesic flow on a manifold with a boundary. Billiards give models for many problems of classical mechanics, statistical physics, optics, and acoustics.

The statistical properties of one particular class of billiards were studied by Sinai [1] who called them dispersing. The simplest example of such billiards is the motion of a material point on a torus with elastic reflections from fixed spherical particles. It was shown in [1] that the corresponding dynamical system is *K*-system. In [2] this result was generalized to a wider class of dispersing billiards.

An intuitive explanation of the analogy between dispersing billiards and goedesic flows in spaces of negative curvature is due to Arnold [3]. The exponential expansion of trajectories, which is due in geodesic flows to negative curvature, follows from collisions with the convex (from inside) components of the boundary of the domain.

Hopf [4] established that a geodesic flow on a surface with the gaussian curvature taking the values of different signs is ergodic if every trajectory spends most of the time in the regions with negative curvature. The analogous result has been proved [5] for domains with boundaries containing both dispersing and focusing components.

In accordance with the traditional ideas stochastic properties of the billiard follow from the scattering of trajectories which results from collisions with the boundary. If small focusing parts of the boundary are also present and the scattering is sufficiently large then the stochasticity is conserved. However, it is

0010-3616/79/0065/0295/$03.60