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A Link Between Scattering Resonances and Dilation Analytic Resonances in Few Body Quantum Mechanics

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Abstract. *We study the (2 cluster)→(2 cluster) scattering amplitudes for classes* of two, three, and four particle dilation analytic Schrödinger operators whose two-body potentials fall off exponentially. As functions of the energy, these amplitudes are shown to have meromorphic continuations on certain Riemann surfaces. We prove that all poles of these continuations are necessarily bound states or dilation analytic resonances [i.e., eigenvalues of $H(\theta)$ for some θ].

1. Introduction

Within the theory of dilation analytic Schrödinger operators there are two very different definitions of resonances. A *dilation analytic resonance* is a complex eigenvalue of an operator $H(\theta)$. *A scattering resonance* is a pole of the analytic continuation of a scattering amplitude. Scattering resonances are directly related to quantities measured in experiments; dilation analytic resonances are easier to estimate by numerical methods.

In this paper a connection between the two definitions is established. We study classes of two, three, and four particle dilation analytic Schrödinger operators, whose two body potentials fall off exponentially (Yukawa potentials are allowed). On certain Riemann surfaces, we construct the meromorphic continuations of the (2 cluster)→(2 cluster) scattering amplitudes. Our main result (Theorem 3.1) is that within the meromorphy domains which we obtain, every scattering resonance is a dilation analytic resonance.

In the literature, previous results of this genre [2, 9, 12] deal only with the two body case. Various subtleties of the different definitions of resonances are discussed in [14] and in the references given there.

The paper is organized as follows: In Sect. 2, we establish notation, state hypotheses on potentials, prove some preliminary lemmas, and recall the (2 cluster)→(2 cluster) T-matrix formula of [8]. Our main result (Theorem 3.1) is

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