Commun. math. Phys. 65, 97–128 (1979)

Communications in
Mathematical
Physics

A Limit Theorem for Turbulent Diffusion

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Abstract. We show under some specific conditions that the formal diffusion approximation for the motion of a particle in a random velocity field is valid.

1. Introduction

Let $V(x)$ be a random velocity field with given statistical characteristics. Let $x(t)$ be the particle trajectories in $\mathbb{R}^d$ satisfying

$$\frac{dx(t)}{dt} = V(x(t)), \qquad x(0) = x. \tag{1.1}$$

The turbulent diffusion problem consists of analyzing the statistical properties of the trajectories $x(t)$ under various hypotheses on the random velocity field $V(x)$. In particular, in many theoretical investigations, [1–3], one wants to find conditions under which the particle trajectories have classical diffusive behaviour and to compute the diffusion and drift coefficients in terms of the statistical properties of the random velocity field V.

We shall consider here this problem in what is perhaps the simplest situation, namely when

$$V(x) = v + \varepsilon F(x), \tag{1.2}$$

where v is a constant nonzero vector representing the mean velocity, $F(x)$ is a given zero-mean stationary random field and $\varepsilon > 0$ is a small dimensionless parameter measuring the size of the fluctuations. More precise conditions are given in the next section. For ε small and t large ($t \sim \varepsilon^{-2}$ when v and F are given independently of ε) $x(t) - vt$ will behave like a diffusion process. This is the content of Theorem 1 in the next section.

By using formal perturbation theory, as in [4] for example, we shall now derive formulas for the diffusion and drift coefficients of the limiting diffusion process. Again, in Theorem 1 we show that these formulas are indeed the correct ones.

0010-3616/79/0065/0097/$06.40