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PCT, Spin and Statistics, and All That for Nonlocal Wightman Fields

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Abstract. It is shown how classical results of axiomatic field theory such as the PCT and spin-statistics theorem may be generalized to nonlocal tempered fields. The method is also applicable to the theory of essentially local non-localizable fields.

1. Introduction and Results

It is known [5] that there are nontrivial models fulfilling all the Wightman axioms (see [6], Chap. 3, the test space has to be modified), except local commutativity. On the other hand, despite tremendous effort nobody has been able to construct nontrivial models in 4-dimensional space-time fulfilling all the Wightman axioms including local commutativity. In this situation it is quite natural to ask why local commutativity is generally believed to be indispensable. Cogent physical arguments are not known—although the idea of macrocausality should be somehow incorporated into the axiomatic scheme, of course. Thus it seems to be its enormous predictive power that makes physicists believe in local commutativity. The C^*-algebraic as well as the Euclidean approach to field theory cannot be imagined without this axiom and the standard proofs of most of the classical results of axiomatic field theory rely on local commutativity.

This does not mean, however, that such important results as the PCT and spin-statistics theorem cannot be proved without local commutativity. Indeed, the purpose of this paper is to stress the contrary.

Let us consider a field theory with the field operators $\varphi_1(x), \dots, \varphi_N(x)$ fulfilling all the Wightman axioms except local commutativity. If g is a tempered test function such that

(A1): the Fourier transform $\tilde{g}$ is positive-valued and[1]

(A2): $g(x)$ as well as all its partial derivatives of arbitrary order vanish stronger than exponentially when $\|x\| \to \infty$

we call two fields $\varphi_j(x), \varphi_k(x)$ *g-asymptotically commuting* (*resp. anti-commuting*)

[1] The symbol $\|\cdot\|$ is used for the norm in finite-dim. Euclidean space as well as in the Hilbert space of states

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