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Decay of Classical Yang-Mills Fields*

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Abstract. The classical Yang-Mills equations in four-dimensional Minkowski space are invariant under the conformal group. The resulting conservation laws are explicitly exhibited in terms of the Cauchy data at a fixed time. In particular, it is shown that, for any finite-energy solution of the Yang-Mills equations, the local energy tends to zero as $t \to \infty$.

1. Introduction

Since the conformal group is 15-dimensional, Noether's theorem implies that there must exist 15 independent conservation laws for the Yang-Mills equations [13]. Ten of them are the familiar laws of conservation of energy, momentum and angular momentum. One is the dilation law due to scale invariance and the remaining four are the inversional laws.

From the first inversional law comes the major decay result. More precisely, for any smooth solution for which the $rF^{\mu\nu}$ are square integrable, the energy within any cone, which expands at a strictly slower speed than that of a light cone, decays to zero at the rate t^{-2} as $t \to \infty$. Moreover, for any finite-energy solution, the energy within such a cone tends to zero as $t \to \infty$. This can be interpreted as stating that all the energy of a solution radiates out along the light cone; that is, at characteristic speed. In particular, there are no "classical lumps". Earlier results asserted that the energy within a fixed sphere tends to zero for some sequence of times $t_n \to \infty$ [1, 4, 11] and that the radius of gyration moves at characteristic speed [2]. The exact analogue of our result was first derived for the linear wave equation in [5, 14] and for the nonlinear wave equation $\Box u = u^3$ in [9].

The conservation laws are exhibited in terms of the Cauchy data at fixed times, as is appropriate in the study of the existence and asymptotic behavior of solutions. An exposition of these ideas in the case of the nonlinear Klein-Gordon equation may be found in [10]. A similar program of deriving conservation laws is

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