Commun. math. Phys. 64, 269—278 (1979)

Communications in
Mathematical
Physics

Universal Metaplectic Structures and Geometric Quantization

Michael Forger and Harald Hess*

Institut für Theoretische Physik, Freie Universität Berlin, D-1000 Berlin 33

Abstract. The recently developed concepts of generalized and universal spin structures are carried over from the orthogonal to the symplectic and unitary cases. It turns out that the analogues of $Spin^c$-structures, namely the Mp^c-structures and MU^c-structures, are sufficient to avoid topological obstructions to their existence. It is indicated how this fact can be used in the geometric quantization of certain suitably polarized symplectic manifolds with arbitrary second Stiefel-Whitney class, where the usual Kostant-Souriau quantization scheme breaks down.

1. Introduction

In field theory, an important criterion for a Riemannian manifold M[1] to be a reasonable model of space-time is that it admit spinors [7]. The conventional method of dealing with this problem is to require that M has a spin structure [15]. The bundle of spinors over M is then the complex vector bundle associated to the corresponding principal bundle of spin frames over M and the spin representation of its structure group *Spin* on the space of spinors S.

On the other hand, in geometric quantization, one can apply the same idea to a symplectic manifold M and require that M has a metaplectic structure. The bundle of symplectic spinors over M is then the complex vector bundle associated to the corresponding principal bundle of metaplectic frames over M and the metaplectic representation of its structure group Mp on the space of symplectic spinors S. In some sense, this contains the complex line bundle of pure symplectic spinors over M, which provides for an explicit realization of the bundle of half-forms and the BKS pairing and hence plays an important role in geometric quantization. For more details, we refer to [14].

---

1 For simplicity, we work with Riemannian $(++\ldots+)$ rather than Lorentzian $(+-\ldots-)$ manifolds M and assume them to be even-dimensional, but this does not affect our arguments concerning spin and spinors. We also tacitly assume M to be oriented (so that in particular, the first Stiefel-Whitney class $w_1(M) \in H^1(M, \mathbb{Z}_2)$ of M vanishes)

* Supported by the Studienstiftung des deutschen Volkes

0010-3616/79/0064/0269/$02.00