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# Characterization and Uniqueness of Distinguished Self-Adjoint Extensions of Dirac Operators

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**Abstract.** Distinguished self-adjoint extensions of Dirac operators are characterized by Nenciu and constructed by means of cut-off potentials by Wüst. In this paper it is shown that the existence and a more explicit characterization of Nenciu's self-adjoint extensions can be obtained as a consequence from results of the cut-off method, that these extensions are the same as the extensions constructed with cut-off potentials and that they are unique in some sense.

In the Hilbert space $H := (L^2(\mathbb{R}^3))^4$ the minimal Dirac operator of a spin $\frac{1}{2}$ particle with non-zero rest mass under the influence of a potential $q:\mathbb{R}^3_+ \to \mathbb{R}$ ($\mathbb{R}^3_+ := \mathbb{R}^3 \setminus \{0\}$) $q$ measurable, is given by

$$T := (\boldsymbol{\alpha}\cdot\mathbf{p} + \beta + q) \upharpoonright D_0 ,$$

$$D_0 := (C_0^\infty(\mathbb{R}^3_+))^4$$

(cf. [2, 6, 10] for more details).

We consider Coulomb like potentials $q$, i.e. potentials $q$ with

$$\mu := \sup_{\mathbb{R}^3_+} |xq(x)| < \infty .$$

Then $T$ is essentially self-adjoint if $\mu < \frac{1}{2}\sqrt{3}$ (cf. [6]) and in general not essentially self-adjoint if $\mu > \frac{1}{2}\sqrt{3}$.

But as long as $\mu < 1$, physically distinguished self-adjoint extensions of $T$ still exist:

By means of cut-off potentials we have shown in [8–10], that for $q$ semibounded from above (or from below) and $\mu < 1$

$$\tilde{T} := T^* \upharpoonright (D(T^*) \cap D(r^{-\frac{1}{2}}))\ ^1$$

is a self-adjoint extension of $T$ (cf. the appendix for not semibounded potentials).

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1 For $\alpha \in \mathbb{R}$ we denote by $r^\alpha$ the closure of the multiplication operator

$$R_\alpha : D_0 \to H, \quad u(x) \to |x|^\alpha u(x) \quad (u \in D_0,\ x \in \mathbb{R}^3_+) .$$

The multiplication operators $q$, $r^\alpha q$ are defined analogously

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