

Uniform Cluster Estimates for Lattice Models

V. A. Malyshev

Moscow State University, 117234 Moscow B-234, USSR

Abstract. In [2] we obtained the complete cluster expansion for transfer-matrix for lattice models with bounded potentials in high-temperature region. Here we obtain necessary cluster estimates for semiinvariants of functionals over Gibbs field with unbounded interaction in high-temperature region and for contour models in low-temperature region.

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Introduction

One of the main obstacles on the way to the proof of asymptotic completeness for $\lambda P(\varphi)_2$ models of quantum field theory for small λ was the absence of N -particle cluster expansion for all N in $0 < \lambda < \lambda_0$. Glimm, Jaffe and Spencer [1] obtained N -particle cluster expansion with $N \rightarrow \infty$ if $\lambda \rightarrow 0$.

In [2] the author obtained the complete cluster expansion (i.e. for all N) in high temperature region $|\beta| < \beta_0$ for lattice fields with bounded interaction potential. The main ingredients in [2] were strong cluster estimates of semiinvariants $\langle F_{B_1}, \dots, F_{B_n} \rangle$ where F_B depends only on values of Gibbs random field in points $\iota \in B$, B is finite.