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Time-Ordered Products and Schwinger Functions

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Abstract. It is shown that every system of time-ordered products for a local field theory determines a related system of Schwinger functions possessing an extended form of Osterwalder-Schrader positivity and that the converse is true provided certain growth conditions are satisfied. This is applied to the φ_3^4 theory and it is shown that the time-ordered functions and S-matrix elements admit the standard perturbation series as asymptotic expansions.

I. Introduction

The present paper is a sequel to [EEF], in which some of the existing models of field theories in 2 space-time dimensions were considered. In order to study the dependence of the S-matrix on the coupling constant, time-ordered functions were constructed in a natural way, by taking essential advantage of the local integrability of the Schwinger functions. (This very property served to define the Schwinger functions as distributions defined everywhere, including coinciding points.) In other models, more singular Schwinger functions occur, and the method of [EEF] cannot be applied. In this paper, a general discussion of the connection between Schwinger functions and time-ordered products is given and applied to the φ_3^4 theory. We show that any Wightman theory equipped with time-ordered products possesses Schwinger functions (considered as distributions over the whole Euclidean world, including coinciding points) which exhibit "extended Osterwalder-Schrader positivity". Conversely, given a set of Schwinger functions possessing this extended positivity together with growth properties similar to those of [OS2], it is possible to supplement the constructions of [OS1, OS2, G1] with a construction of time-ordered products, in a canonical manner. Finally we consider the model φ_3^4 and, starting from results accumulated in the literature [G2, GJ, Fe, FO, MS1, MS2, B, FR, C], we extend to this model the analysis of [EEF], showing in particular that the time-ordered functions and the S-matrix

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