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Relativistic Models of Nonlinear Quantum Mechanics

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Abstract. I present and discuss a class of nonlinear quantum-theory models, based on simple relativistic field theories, in which the parameters depend on the state of the system via expectation values of local functions of the fields.

1. Introduction

The linearity of quantum mechanics, expressed in the "superposition principle" is anomalous. Linearity is a common feature of physical theories, but in all other known cases it is an approximation. The range over which linearity holds may be extensive, but is always limited: Maxwell's equations break down for very intense fields (when pair creation is important) and the linearity of space-time itself is a weak-field approximation.

Hitherto all the specific nonlinear generalizations of quantum theory that have been proposed have been nonrelativistic, or at best one-particle relativistic theories [1, 2]. However, if there is a nonlinear theory underlying quantum mechanics, it seems likely that when the nonlinearity is important interparticle interaction, and particle creation and annihilation, are important too. Thus we should really be seeking a nonlinear generalization not of a one-particle Schrödinger equation but of a quantum field theory.

In this paper, I present a class of nonlinear quantum models based on simple field theories, and discuss their properties. It appears that these models, and some nonrelativistic ones, do not fit neatly into the formal schemes that have been set up to accommodate generalizations of quantum mechanics, such as the "convexity" formalism [1, 3–7] based on the operational approach of Haag and Kastler [8]. I shall indicate some of the problems in this paper, and discuss a new formal structure in a later publication.

Quantum mechanics is among the most successful of physical theories. Many of its predictions, especially those of quantum electrodynamics, have been verified with unparalleled precision. So it may seem foolhardy to question its most basic tenet. On the other hand, despite its successes, the interpretation of quantum mechanics remains problematic [9]. There is still no generally accepted solution to the problem of "reduction of the wave packet". Moreover, there are grave difficulties confronting relativistic quantum theory, difficulties which have been

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