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Absence of Symmetry Breakdown and Uniqueness of the Vacuum for Multicomponent Field Theories

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Abstract. Correlation inequalities are used to show that the two component $\lambda(\phi^2)^2$ model (with HD, D, HP, P boundary conditions) has a unique vacuum if the field does not develop a non-zero expectation value. It follows by a generalized Coleman theorem that in two space-time dimensions the vacuum is unique for all values of the coupling constant. In three space-time dimensions the vacuum is unique below the critical coupling constant.

For the n-component $P(|\phi|^2)_2+\mu\phi_1$ model, absence of continuous symmetry breaking, as μ goes to zero, is proven for all states which are translation invariant, satisfy the spectral condition, and are weak* limit points of finite volume states satisfying N_{loc}^{τ} and higher order estimates.

I. Introduction

It is a general fact in statistical mechanics and quantum field theory that the appearance of multiple phases and spontaneous symmetry breaking occurs more readily as the number of space dimensions increases. In the case of the statistical mechanics of lattice systems with a continuous internal symmetry group, spontaneous symmetry breakdown can occur for three dimensional lattices [10] while for two dimensional lattices every equilibrium state is invariant under the internal symmetry [4]. For the two component rotator, absence of symmetry breakdown implies uniqueness of the phase [2].

Multicomponent quantum field theories can exhibit spontaneous symmetry breakdown in three space-time dimensions [10], while in two space-time dimensions, a general result due to Coleman [3, 11] "There are no Goldstone bosons in two space-time dimensions" shows that spontaneous breakdown of a continuous internal symmetry cannot occur, provided the symmetry is generated by a local conserved current. Given these results, we have considered two questions. The first is whether absence of symmetry breakdown implies uniqueness of the vacuum

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